A new methodology for cup wheel precision grinding of rotational quadric surface

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Abstract. This study proposed a novel methodology called Equivalent Sphere Swing Evolution Grinding (ESSEG) for cup wheel grinding of rotational quadric surface in order to improve the machining efficiency and simplify the complex requirements of machine tools. The mathematical model of grinding process was concluded to verify the feasibility of precision grinding of rotational quadric surface based on the new method. The mathematical model of wheel pose was established and the grinding process was proved to be of non-interference in a wide range of processing parameters. It is revealed that the wheel pose model is not unique in grinding the certain rotating surface. The numerical controlled trajectory parameters were mathematically modelled and a stepwise grinding model was presented for experimentally grinding of the concave paraboloid. The experiment was designed and conducted on the self-developed grinding system. The results indicated that the proposed new methodology can efficiently finish the grinding of large scale rotational surface with the expected profile accuracy, which can be further improved by increasing the accuracy of machine tool and optimization of grinding process parameters.

1. Introduction

The development of astronomical telescope, satellite optical system, space detection and energy engineering highly demands for components with spherical, aspheric and free form surfaces of high form accuracy. Numerous researches have been conducted to develop machining methodologies and technologies for manufacturing such components [1]. Among them, the axisymmetrical aspheric surface components made of ceramic materials such as silicon carbide has been extensively used as lenses, mirrors, molding dies due to their special optical functions [2]. They have excellent mechanical and thermal properties as well as good chemical stability, but are also difficult to be fabricated due to their high hardness and brittleness. The machining process of the aspheric surface generally consists of precision grinding or turning, lapping and polishing. Among them, precision grinding is usually the critical process. It can reduce the machining time, as well as improve the machining accuracy and surface quality, which enhances the efficiency of the subsequent process such as lapping or polishing [3]. Many research efforts have been directed towards precision machining of symmetrical aspheric surface, and the grinding methods such as cross grinding, parallel
grinding, single-point inclined axis grinding have been developed [2, 4]. But it is still a challenge to develop high precision and high efficiency machining technologies for manufacturing such large scale components.

Numerous researches on the methodologies for grinding of aspherical surface were conducted [5]. Hinn et al. [6] presented an efficient device for grinding and polishing of the asphere. Both cup-wheel and peripheral grinding wheel spindles are incorporated into one platform. The cup-wheel grinding is advantageous for spherical grinding with high efficiency, while the peripheral wheel is used for aspherical grinding to perform the point grinding of the revolving surface. In order to reduce the influence of wheel profiles error on form accuracy of the workpiece, Xie et al [7] proposed a dispersed grinding wheel profiles to replace the ideal one for planning 3D tool path. But the wheel wear development during the grinding process was not considered. Yin et al [8] presented an investigation on the 3D off-aspheric surface generation in single point diamond turning using slow tool servo. Huo et al. [9] developed a method for machining the rotationally symmetric surface by infeed grinding with a rotary table and a cup wheel. It can be used to precisely generate the convex or concave conical surface in a cost effective way. Zhang [10] developed an ultrasonic vibration-assisted five-axis fix-point grinding technology to machine the large silicon carbide mirror blank for the space-based telescope. Enomoto et al. [11] found that traversing a wheel outward from the workpiece center allows grindability to be almost constant in axisymmetrical grinding which helps to obtain high form accuracy. A grate parallel grinding method was proposed for surface grinding of large axisymmetric aspheric lenses [12]. The advantage is the grinding point was changed along the wheel profile and the wheel wear decreased remarkably to obtain better form accuracy of the ground aspherical surface. But the grinding parameters should be appropriately selected to avoid the waviness error. Xie, et al. [13] proposed a method for 5-axis precision grinding of glass freeform surface without on-machine wheel-profile truing by using a self-dressing diamond wheel. It was a point grinding with a virtual ball-end algorithm along with a fixed tool posture angle.

To sum up, the grinding method, wheel wear, tool setting error all greatly affect the form accuracy and grinding efficiency of the aspheric surface, especially the large surface. This paper proposes an ESSEG (Equivalent Sphere Swing Evolution Grinding) based novel methodology for cup wheel grinding of symmetrical aspheric surface. A novel grinding methodology called ESSEG based grinding is proposed for cup wheel grinding of large symmetrical aspheric surface and the mathematical model was deduced to verify the kinematical feasibility. In contrast to traditional cup wheel grinding of aspheric surface, the new method uses a multipoint contact kinematics and the grinding requires only three basic movements and offers high dynamic stiffness. The wheel wear is uniform across the wheel end face due to the identical abrasive working condition and the material removal rate is improved with minimal tool wear due to the grinding mechanism.

The remainder of this article is organized as follows. In section 2, the methodology of ESSEG based aspherical surface grinding is proposed. The cup wheel posture and position modelling is discussed in section 3. In section 4, mathematical modelling of control parameters is introduced. Section 5 introduces the design of a large aspherical surface grinding system based on the presented method. In section 6, the grinding experiment is conducted and the
results are discussed.

2 ESSEG based aspheric surface grinding

A quadric curve in plane XOY rotates around axis Y to perform an axisymmetric aspherical surface in workpiece coordinate system O-XYZ as shown in Fig. 1. The quadric curve is called generating line of the axisymmetric surface and axis Y is called the rotation axis of the surface. Assume the generating curve is parabola, then the performed surface is axisymmetric paraboloid. Based on the axial symmetry of the surface, the entire paraboloid can be processed by machining every point of the generating line of the axisymmetric surface.

The Schematic of ESSEG based grinding method is shown in Fig. 1. The paraboloid workpiece rotates around axis Y at low velocity \( n_1 \) in workpiece coordinate system. The manufacture coordinates \( O' - X'Y'Z' \) is created based on the movable worktable on which the wheel spindle system is set up. The coordinates \( O - X'Y'Z' \) is movable and parallel to workpiece coordinates. The worktable can move along axis X or Y in plane XOY. The wheel spindle can circles axis \( Y' \) to do swinging in coordinate plane \( X'O'Y' \). \( O' \) is the swinging point as well as the coordinate origin. The spindle center line intersects with axis \( Y' \) at the included angle \( \alpha \). The distance from \( O \) to the end face of cup wheel is \( L \). The radius of cup wheel is \( R \) and the wheel rotates at high speed \( n_2 \) while grinding. In contrast to traditional cup wheel grinding of aspheric surface, the new method uses a multipoint contact or full line contact kinematics. And the grinding requires only three basic movements that are the movements of worktable along axis X and Y and the swinging movement of wheel spindle respectively. The fewer-axis structure offers high dynamic stiffness.

It can be proved that the cup wheel can finish grinding the entire quadric surface without tool-workpiece interference under certain conditions. The translation of the worktable enables the dynamic change of equivalent swinging radius of wheel spindle. In the meantime, it reduces the demand for large spindle swinging radius, i.e. shortens the physical swinging radius of wheel spindle, which solves the great challenge that the large spindle swinging radius will reduce the structure rigidity while grinding the large axisymmetric surface.
The center line of wheel spindle is in plane XOY. The cup wheel rotates around the centerline while the workpiece rotates around axis Y. The resultant movement enable the wheel to perform a machining area shown as the mesh region in Fig. 2. The machining area is essentially a spherical segment, and the corresponding sphere is defined as the equivalent grinding sphere. The spherical segment performs a section of grinding arc as the machining profile in the ground surface in plane XOY. The equivalent grinding sphere can be controlled to swing along the generatrix of the axisymmetric surface, and the produced grinding arcs perform the envelope of the generatrix which can finally finish the machining of the aspheric surface. The grinding method is called equivalent sphere swing evolution grinding (ESSEG) method.

The center of equivalent grinding sphere $O'$ is the intersection point of axis Y and spindle centerline, which is also the normal through the end face center of cup wheel. The radius of equivalent sphere is $R'$, which is the distance between sphere center and arbitrary point at the grinding circle arc. Suppose $L_1$ is the distance between $O'$ and the center of wheel end face. The equivalent sphere radius can be calculated as Eq. (1).

$$ R' = \sqrt{L_1^2 + R^2} $$  \hspace{1cm} (1)

It can be proved that both equivalent sphere center $O'$ and radius $R'$ determine one grinding point in the generatrix, and also have but not the only mapping relation with the position and posture of grinding wheel. By control of the pose and position of cup wheel, the grinding circle arc of the equivalent sphere can envelop the generatrix and eventually finish the processing of aspheric rotary surface.

**3 Cup wheel posture and position modelling**

**3.1 Non-interference proof of ESSEG based cup wheel grinding**

Assume the generatrix equation of axisymmetric surface is as Eq. (2).

$$ y - f(x) = 0 $$ \hspace{1cm} (2)

The curvature radius $K$ of one point on generatrix can be represented as

$$ K = \frac{y''}{(1 + y'^2)^{3/2}} $$ \hspace{1cm} (3)

Assume a normal of point A on generatrix intersects with axis Y at point B. The distance between point A and B is defined as the equivalent grinding sphere radius $R'$, which can be calculated as

$$ R' = \frac{x}{y} \sqrt{1 + y'^2} $$ \hspace{1cm} (4)

In order to meet the requirement of noninterference grinding, the following condition should be satisfied as equation (5).

$$ \frac{x}{y} \sqrt{1 + y'^2} \leq \frac{(1 + y'^2)^{3/2}}{y''} $$ \hspace{1cm} (5)

It can be proved that the common quadric surface including revolution paraboloid, hyperboloid and ellipsoid can all satisfy the demand of equation (5). It can be concluded that
the ESSEG based grinding method can be applied in grinding common quadric surface without interference between tools and workpiece.

3.2 Control of wheel position and posture
The wheel position and posture should be automatically controlled all the time while grinding the aspherical surface using the ESSEG based method. The equivalent sphere envelops the generatrix to perform the rotary surface. As shown in Fig. 3, the generatrix equation of the machined surface can be expressed as

\[ y - f(x) = 0 \]  
\[ A(x,y) \] is the current grinding point on the generatrix. \( C(0, y_0) \) is the center of equivalent grinding sphere. \( B(x_1, y_1) \) is the center point of end surface of cup wheel. \( \alpha \) is the angle between the center line of wheel spindle and axis Y and it’s also called inclination angle of the cup wheel. The geometric relationship in coordinate system \( XOY \) can be concluded by following equations.

\[ y_0 = y + \frac{x}{y'} \]  
\[ L_1 = \sqrt{R'^2 - R^2} \]  
\[ x_1 = L_1 \times \sin \alpha \]  
\[ y_1 = y_0 \times L_1 \times \cos \alpha \]  

(9)

It can be derived from Eq. (6) to Eq. (9) that there is only one equivalent grinding sphere corresponding to one given grinding point on the generatrix. But the posture parameters of \( \alpha, x_1, y_1 \) is not unique according to one equivalent sphere. That is, there theoretically exist numerous sorts of wheel inclination angle \( \alpha \) and position of center point \( B(x_1, y_1) \) which determine the pose of wheel while machining any one grinding point on the generatrix.

The theoretical material removal rate (MRR) varies with the change of wheel posture, which also cause the change of wheel force dynamic conditions. There exists a wheel inclination angle which ensures the parallel of wheel end face with the tangential of the generatrix at point A. It performs the condition for the theoretical MRR to reach maximum and better force uniformity while grinding point A in the case of certain cutting depth. The less cutting depth can also be achieved under the condition of same MRR in such case.

Therefore, the wheel posture and position can be controlled during the grinding process to ensure the parallel of wheel end face with the tangential of the generatrix at the grinding point. Assume \( A(x,y) \) is the current grinding point, and as shown in Fig. 3, the posture parameters of \( \alpha, x_1, y_1 \) should satisfy the following equations.

\[ \alpha = \arctan y' \]  

(10)
\[ x_i = \sqrt{\frac{x^2(1+y'^2)}{y'^2} - R^2 \times \frac{y'}{\sqrt{1+y'^2}}} \]  
(11)

\[ y_i = y + \frac{x}{y'} \sqrt{\frac{x^2(1+y'^2)}{y'^2} - R^2 \times \frac{1}{\sqrt{1+y'^2}}} \]  
(12)

The cup wheel posture and position can be precision controlled to meet the modeling requirements of Eq. (10) to Eq. (12) by controlling the position of origin \( O'(X,Y) \) of manufacture coordinates and the swing angle \( \alpha \) of wheel spindle in the plane XOY of coordinates \( O-XYZ \).

Assuming grinding a given point \( A(x,y) \) on the generatrix, \( \alpha \) should also satisfy Eq. (10), and the control parameters \( X, Y \) should satisfy

\[ X + L \times \sin \alpha = x_i \]  
(13)

\[ Y - L \times \cos \alpha = y_i \]  
(14)

Combine Eq. (13) and (14) with Eq. (11) and (12), it can be derived that

\[ X = \sqrt{\frac{x^2(1+y'^2)}{y'^2} - R^2 \times \frac{y'}{\sqrt{1+y'^2}}} - L \times \frac{y'}{\sqrt{1+y'^2}} \]  
(15)

\[ Y = y + \frac{x}{y'} \sqrt{\frac{x^2(1+y'^2)}{y'^2} - R^2 \times \frac{1}{\sqrt{1+y'^2}}} + L \times \frac{1}{\sqrt{1+y'^2}} \]  
(16)

Assuming the paraboloid is to be ground, the generatrix equation is

\[ f(x) = px^2 \]  
(17)

5 Experiment and discussion

5.1 Experimental setup

Based on the presented ESSEG based grinding principle, a prototype machine for precision grinding of large revolution aspherical surface is developed as shown in Fig. 4. The SiC workpiece with parabola surface was mounted on the B turntable. The generatrix equation is \( y = 0.0016x^2 \). The outer diameter is 420mm, the motorized wheel spindle power is 22kW and electric power of the servo motors for axis X, Y and C is 2kW respectively. The cup wheel is made up of four cuboid abrasive blocks evenly glued to the rim part of end face of a wheel support. Such cup wheel structure is in favor of chip removal and cooling in grinding process. The vitrified bond abrasive block is
comprised of diamond abrasive with 100 ANSI mesh size. In grinding process, the workpiece rotated in speed of 10rpm. The grinding wheel speed is 12000 rpm. The feed speed of wheel is 2mm/min. Based on the hierarchical grinding model, the hierarchy interval is 0.02mm in Y direction.

5.2 Result and discussion

In this study, a 3D laser scanning measurement is introduced to measure the ground inner surface. It is named AMETEK handy CREAFORM 3D laser scanner HandySCAN 700. The scanning area of single sweep is 275mm×250mm. The light source consists of seven beams of cross laser ray. The measurement resolution is 0.05mm and the maximal precision is up to 0.03mm. The measured depth of field is 250mm. The spatial information of finished surface can be data collected by multi-zone and multi-scanning mode. The measure test site is shown in Fig. 8. After finishing scanning, the measured 3D surface data can be merged to acquire the integrated spatial information of machined surface. The laser scanning picture is shown in Fig. 5. The point cloud data can be converted into standard space coordinates which can be offered to further data processing.

The effective point cloud data are within the range between Z=30mm and Z=165mm, which counts up to be 209799. The paraboloid equation can be fitted using least square method based on the effective point cloud. The fitted result is represented as Eq. (18) and the fitted surface is shown in Fig. 6.

\[ y = 0.0016x^2 + 0.0016z^2 - 0.0025x + 0.0002z - 59.5963 \] (18)

Compared with theoretical parabolic equation \( y = 0.0016x^2 \), the surface shape parameter keeps the same as the fitted surface. It verifies the proposed ESSEG grinding method in grinding the aspheric surface and also indicates that the surface-fitting algorithm can be used in analyzing of surface shape error.

The clusters of data were extracted depending on the certain interval of coordinates of axis Y. The normal distances between the points of each cluster and the fitted parabolic surface were calculated, and the profile error of the ground surface can be analyzed. In this study, the points were selected in
the range of y-[-15, -58] and 17 clusters were performed on the condition that the interval of y coordinate between adjacent clusters is 2.5mm. The deviation range of coordinates y is between -0.05mm and 0.05mm for each cluster.

The normal distance between every point in each cluster and the fitted paraboloid was calculated. The maximal profile error for each cluster is less than 0.02mm. The maximum profile error for all clusters is less than 0.089mm and the average is less than 0.058mm. It is also observed that the profile error has little change for the area where Y coordinate value is less than 70% of paraboloid height. The profile error is positively correlated with Y coordinate value when it is greater than 70% of paraboloid height. The experiment results indicate that the proposed method is feasible and effective for cup wheel precision grinding of large scale rotational surface.

6 Conclusions

(1) This study proposed a ESSEG based grinding methodology for cup wheel grinding of large symmetrical aspheric surface. Two linear and one rotary movements are enough for precision grinding of the aspheric surface, which remarkably simplified the complex of machine tools and offered high dynamic stiffness as well.

(2) The posture model of cup wheel grinding of quadratic aspheric surface was established based on the proposed grinding method. It is proved that non-interference ESSEG based grinding of quadric surface can be achieved without principle errors. The cup wheel is proved to have diversity of postures while grinding a specific aspheric surface and the optimized posture can be obtained on the condition of the wheel end face paralleling with the tangential of the generatrix at the grinding point.

(3) Based on the ESSEG based grinding methodology, a prototype machine tools for precision grinding of large revolution aspherical surface is developed and the grinding process is modelled. The processing scope can be enlarged by reducing the radius of grinding wheel and the corresponding expression of quantitative characterization is deduced.

(4) The prototype machine tool was developed based on the proposed ESSEG methodology and the experiment of aspherical surface grinding was conducted. The three dimensional laser scanner was used for measurement of form accuracy. The results reveal the distribution of profile errors for different area of ground surface and verify the effectiveness of the proposed ESSED based method for grinding of symmetrical aspherical surface.

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References


