Modeling of the micro-grinding process considering the grinding tool topography
Mohammadali Kadivar¹,²,a, *, Ali Zahedi¹,b, Bahman Azarhoushang¹,c, and Peter Krajnik²,d

¹ Institute for Precision Machining (KSF), Hochschule Furtwangen University, Jakob-Kienzle-Str.17, 78054, Villingen-Schwenningen, Germany
² Department of Materials & Manufacturing Technology, Chalmers University of Technology, Hörsalsvägen 7B, 412 96 Gothenburg, Sweden

aKamo@hs-furtwangen.de, b al.zahedi@hs-furtwangen.de, caza@hs-furtwangen.de, dpeter.krajnik@chalmers.se

Keywords: Single-grain interaction; micro grinding; diamond grinding pin; grinding pin topology.

Abstract. The micro topography of the grinding tool has a considerable influence on the cutting forces and temperature as well as the tool wear. This paper addresses an analytical modeling of the micro-grinding process based on the real tool topography and kinematic modeling of the cutting-edge-workpiece interactions. An approximate shape of the abrasive grains and their distribution is obtained from the confocal images, which are taken from the tool surface – determining the grain height protrusion and the probability density function of the grains. To determine the grinding forces, a transient kinematic approach is developed. In this method, the individual grit interaction with the workpiece is extended to the whole cutting zone in the peripheral flank grinding operation. Hence a predictive model of cutting forces and surface roughness in micro grinding of titanium grade 5 is developed. Finally, the simulated forces and surface roughness are validated by the experimental results.

Introduction

The trend towards miniaturization poses a lot of challenges to the production technology in many areas in the industry. A reliable and cost-effective way of production for small geometries mainly with micro-structured surfaces and high dimensional accuracy and proper surface quality is demanded. Micro-machining operations, such as micro-grinding, milling, and turning, are distinguished from other non-mechanical operations like lithography and are able to produce the micro-parts with high flexibility. They are a suitable choice for small batch production as well [1]. Among the micro manufacturing operations, micro-grinding is able to produce parts with micro features and superior surface quality and outperforms micro-milling [2]. Micro-grinding significantly differs from conventional grinding process due to the size effect. The diameter of the grinding tool in the micro-grinding operation is usually less than 2 mm. The quality of the micro-parts is highly affected by the process parameters, micro-grinding tool specifications, and the microstructure of the workpiece material [3].

Limited researches, which are dealing with the fundamental understanding of the material removal mechanisms in the micro-scale (single grain-workpiece interaction), are available [4–6]. Moreover, the micro-scale numerical modeling methods are developed to describe the plastic behavior of the workpiece material at the high temperature and strain-rates linked with the grinding process [7]. Zahedi and Azarhoushang [8] simulated the interaction of cBN cutting grains with a bearing steel workpiece using Finite Element Method (FEM) and showed that the calculated forces are qualitatively in accordance with the experimental data. However, the error between calculated and experimental forces was relatively high. Cheng et al [9] presented a mathematical model for the prediction of the micro drill-grinding force. Park and Lian [3] modeled the micro-grinding forces based on the physical analysis of the process. The single grit interaction, heat transfer behavior, and
micro-grinding wheel topography were included in their model. Cheng and Gong [10] investigated the effect of the undeformed chip thickness on the micro-grinding of the single crystal silicon.

The number of research papers in the field of micro-grinding process are just limited to study of the grindability of the various material and there is lack of modeling studies in the field of micro grinding. In this paper, the experimental results for single grain scratch test for titanium material have been used from the work of Feng and Cai [11]. Based on the experimental data a regression model has been extracted for the normal grain force and pile-up characteristics as a function of the grain size, the cutting speed, and the depth of cut. The results for single-grain scratch, i.e. the force values and the chipping mechanism have been further extended to the aggregate action of the cutting grains in the grinding process through the kinematics of the process. The simulated forces and surface roughness have also been validated by the experimental results in micro-grinding of titanium grade 5. The results showed that the developed model is able to predict the cutting forces and surface roughness with an acceptable accuracy.

1. Modeling of the micro-grinding process

1.1. Single grain scratch.

The experimental results for single grain scratch test of the titanium alloy were extracted from Feng and Cai work, where experiments were performed at different cutting speeds and depth of cuts [11]. The results for both grinding forces ($F_n$ and $F_t$) correspond to different grain cross-section areas ($A$) have been given in Fig. 1. The cross-section area of grain and workpiece engagement could be directly related to the depth of cut and the average grain size (assuming that the tip of the abrasive grain has a spherical form).

![Fig. 1. Single-grit grinding forces a) normal grinding force b) tangential grinding force [11]](image)

Based on the experimental results, a force equation (a regression model) as a function of the average grain size, the cutting speed, and the depth of cut is developed. The unknown coefficients in the model are defined by minimizing the square of the deviations from the experimental results (least
The regression equation for the normal grain force can be accordingly expressed as:

\[
F_n = 4.65 \times 10^{-5} d_g^{1.36} a_g^{0.513} v_c^{-0.082}
\]  

(1)

where \(d_g\) is the average grain size, \(a_g\) is the grain depth of cut in \(\mu m\), and \(v_c\) is the cutting velocity in \(m/s\). The above equation implies the rise of the cutting force with increasing the grain size and depth of cut; and reduction of the cutting force with increasing the cutting velocity. It is valid for the scratch tests of titanium for the range of performed experimental parameters. The latter reflects the dependency of the material properties on the strain-rate. According to this regression equation, the cutting conditions of each cutting grain can be expressed in the term of normal cutting force. Through integration of the normal grain force components of individual grains in tool-workpiece contact zone of the actual grinding process, the total normal grinding force can be calculated. The tangential force component can be further obtained by applying the grinding force ratio (ratio of the tangential to the normal grinding forces = 0.65 for the titanium) [13].

![Fig. 2. a) Confocal picture from the grinding tool, b) Profile of the section 1, c) Profile of the section 2.](image)

1.2. Grinding tool modeling
The abrasive grains of the modeled grinding tool are distributed according to their real position on the actual grinding tool. For this purpose, several confocal pictures from the surface of the grinding tool were taken. Fig. 2 shows the confocal picture of the actual topography of a vitrified-bonded diamond grinding tool (D45-46-C150) with diameter of 2 mm. After taking the pictures, several sections were extracted and the position and protrusion height distribution of each single grain were extracted and used to model the grinding tool. The height of the grains are determined according to an approach presented in [8] based on a Gamma distribution function. The scale and shape parameters of the distribution function are accordingly defined to fit the confocal microscopy of the extracted sections. Fig. 3 shows a section from the virtual grinding tool modeled according to the actual position of the grains. The whole grinding tool surface is obtained by extending the modelled section of the wheel surface to its whole width and periphery. According to Shaw [14], it is assumed that the tips of the grains have a spherical shape.

1.3. Kinematics of the peripheral flank-surface grinding

The results of the single-grain scratch test can be extended to the aggregate cutting action of abrasive grains on the grinding tool surface through defining the distribution of the grains and the grain-workpiece interaction. By expressing the coordinates of the cutting grains \((X_i, Y_i, Z_i)\) and the individual points on the workpiece surface \((X_j, Y_j, Z_j)\) in a global coordinate system attached to the workpiece, the grain-workpiece engagement can be characterized and interpreted according to the generated regression model. The engagement criteria of the \(i^{th}\) grain can be consequently expressed as:

\[
(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 < \left(\frac{d_i}{2}\right)^2
\]

which \(d_i\) is the diameter of the \(i^{th}\) simulated grain. The equation expresses the instantaneous distance between the points on the workpiece surface and individual cutting grains. In the case of engagement (when Eq. 2 holds), the grinding force components and the workpiece surface topography are calculated according to the engagement depth, grain size and cutting velocity. The proposed procedure is applied to the all grains on the tool-workpiece contact zone at each time step.

The proposed model makes it possible to consider the effects of the individual grain regarding the time-dependent workpiece surface topography modified by previous cutting grain. Therefore, after
the calculation of the grain engagements (Eq. 2) and corresponding forces, the workpiece surface is also modified accordingly. This procedure results in the generation of the workpiece surface profile influenced by all cutting grains throughout the grinding process. A Matlab code was specifically generated for the modelling of tool surface and the time-dependent simulation of the grinding process (force and workpiece surface).

2. Results and validation

2.1. Micro-grinding forces

To verify the simulation results, micro-grinding tests were performed on the Ti4V6Al titanium alloy in the form of a block with the dimensions of 30x20x10 mm by a high precision CNC machine center (Kern Pyramid-Nano). Grinding oil was chosen as lubricant throughout the experiments. The surface roughness was measured by using a surface roughness tester Hommel-Werke model T-1000. Grinding conditions are listed in Table 1. Fig. 4 shows the experimental setup. In all figures dash lines and continuous lines are corresponding to simulated and experimental forces, respectively. Before starting the experimental test the micro-grinding tool was dressed by using a diamond dressing roll with the diameter of 100 mm. The grinding tool was redressed after changing the grinding cutting speed with the new cutting speed.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting Speed (v_c) (m/s)</td>
<td>6, 10 and 14</td>
</tr>
<tr>
<td>Feed Rate (v_f) (mm/min)</td>
<td>500 and 1000</td>
</tr>
<tr>
<td>Depth of Cut (a_e) (µm)</td>
<td>4, 7 and 10</td>
</tr>
<tr>
<td>Dressing Depth of Cut (a_{ed}) (µm)</td>
<td>2</td>
</tr>
<tr>
<td>Dressing Velocity Ratio (q_d)</td>
<td>0.8</td>
</tr>
<tr>
<td>Dressing Feed Rate (v_{fd}) (mm/min)</td>
<td>200</td>
</tr>
</tbody>
</table>

The experimental and simulated normal and tangential grinding forces for different cutting speeds are presented in Fig. 5. As it can be seen in this Fig., simulated and experimental forces follow the same trend and are matched qualitatively. By increasing the cutting speed, the grinding forces decreased in both cases. The kinematic energy and the impact force of each grain increase with rising the cutting speed. Increasing the cutting speed reduces the number of kinematic cutting edges and the mean uncut chip thickness of each active grain.

Fig. 4. a) Experimental set-up and b) Schematic of the grinding process
The number of kinematic cutting edges can be defined by Eq. 3, where \( L_n \) is the spacing along the tool surface from the prospective cutting point to the proceeding active one, \( \delta_n \) is the protrusion deference between the cutting points of \( n \) and \( n-1 \), \( v_w \) is the workpiece velocity, and \( d_e \) is the equivalent diameter of the grinding tool [15]. Decreasing the \( v_w/v_c \) ratio decreases the number of kinematic cutting edges owing to the reduction of grains which do not come into the contact with the workpiece surface. The uncut chip thickness \( (h_m) \) can be defined according to Eq. 4 [15]. However, increasing the \( L \) increases the uncut chip thickness, its effect is smaller than the \( v_w/v_c \) ratio [15]. Therefore, increasing the cutting speed decreases both the uncut chip thickness and the number of kinematic cutting edges. Hence, grinding forces, which are acting on each active grain, are decreased, resulting to lower grinding forces. In Fig. 5, the simulation results for tangential and normal forces were accurate within the total error of 11.5 and 10.5 percent, respectively.

\[
\frac{\delta_n}{L_n} > 2 \left( \frac{v_w}{v_c} \right) \left( \frac{a_e}{d_e} \right)^{1/2} \tag{3}
\]

\[
h_m = 2L \left( \frac{v_w}{v_c} \right) \left( \frac{a_e}{d_e} \right)^{1/2} \tag{4}
\]

**Grinding Parameters:**
- \( v_c = 6,10 \) and 14 m/s
- \( v_w = 500 \) mm/min
- \( a_e = 7 \) μm

**Grinding Tool:**
- D46 C150 V

**Coolant:**
- Oil

**Workpiece:**
- Titanium Grade 5

![Fig. 5. Grinding forces versus cutting speed.](image)

Fig. 6 shows the tangential and normal grinding forces versus the depth of cut. Both tangential and normal forces increased with increasing the depth of cut. The same trend can be seen for simulated force. Higher depth of cut causes higher material removal rate and thicker uncut chip thickness, resulting higher normal and tangential forces. In addition to the thicker uncut chip thickness, increasing depth of cut increases the contact length of the grinding tool with the workpiece and the number of the momentarily engaged cutting edges, inducing higher grinding forces [16]. According to the experimental results, the grinding force ratio lies within 10% of the assumed value in the modelling, therefore, the assumption of a constant force ratio could be acceptable for a specific material and defined range of the grinding conditions.
Although, the prediction of the tangential force is qualitatively matched with those in the experimental study by increasing the depth of cut, the predictions are more accurate at the smaller depths of cut, compared to higher depths of cut. The variation in the predicted grinding forces can be associated with the following causes:

- During the grinding process, the number of active cutting edges changes due to the grain breakage and pull-out. Hence, different active grains are available at different height levels, inducing different force values.
- The actual grains are normally sharper than the assumed spheres.
- In the modeling, it was assumed that the grains are ideally spherical. However, in the real grinding tool they have different shapes.
- The wear of active grains in the real grinding process increases at higher depths of cut. Hence the grit protrusion will be reduced and become smaller than the simulated grit protrusion.

### 2.2. Surface roughness

Fig. 7 shows the simulated and experimental values of the surface roughness versus different depth of cuts. In both simulation and experiment, increasing the depth of cut resulted in the rougher surface finish. As it can be seen in Fig. 7-a, the surface roughness values, predicted by the simulation method, follow the same trend as the experimental values.
Increasing the depth of cut increases the number of kinematic cutting edges and the mean uncut chip thickness of each active grain (Eqs. 3 and 4), therefore it leads to higher values of the surface roughness [15]. On the other hand, increasing the depth of cut increases the grinding forces, consequently increases the stress on the finished surface which causes higher vibration and rougher surface roughness.

The influence of the grinding feed rate on the surface roughness at the cutting speed of 14m/s has been shown in Fig. 8. The grinding feed rate has a negligible effect on the surface roughness. Although the surface roughness predicted by the simulation are finer than those obtained from the experimental study at the cutting speed of 14m/s, the trend of the roughness changes with varying the feed rate. The differences between the simulated and real surface roughness, seen in Fig. 8, can be due to the unavoidable vibration in the grinding process by using the cutting speed of 14m/s. The
tool vibration is one of the influential factors which may significantly influence the surface quality. The very small diameter of the tool along with its high rotational speed causes the tool vibration which has been neglected in the simulation [17].

The simulation results were matched with the experimental results in greater degree at the low cutting speed of 6m/s (Fig. 7), rather than the higher cutting speed (14 m/s) as seen in Fig. 8. For instance, the predicted surface roughness at the cutting speed of 6 m/s, feed rate 100 mm/min, and depth of cut of 7 µm was similar to the real surface roughness (Fig. 7). However, changing the cutting speed from 6 to 14, not only changed the real surface roughness from $R_s=0.89$ to $R_s=2.2$, but also the simulated surface roughness was considerably different from that in the real situation at the cutting speed of 14 m/s. As known in the macro-grinding operations, increasing the cutting speed causes finer surface roughness because of the smaller chip thickness. With comparing Fig. 7 and 8, the simulation model predicted a finer surface roughness with increasing the cutting speed from 6 to 14m/s due to the smaller chip thickness. This does not agree with the experimental results in these figures since the tool diameter is very small and tool vibration occurs at the high rotational speeds which is neglected in the simulation process. Fig. 9 shows the topography of the ground surface according to the simulation study. The initial (non-ground) surface of the workpiece can be distinguished from the ground surface.

![Ground Surface](image)

Fig. 9. Pattern of the ground surface obtained from the simulation model ($v_c=6$ m/s, $v_f=1000$ mm/min, and $a_e=7$ µm)

3. Conclusion

In this paper, the experimental results of the single-grain test of titanium alloy was used to simulate the micro-grinding process with a diamond grinding tool. Based on the confocal measurements of the tool surface, the probability density function of the stochastic distribution of the diamond grains was defined. A virtual micro-grinding tool model was generated based on the actual tool topography. The model considered an approximate shape of abrasive grains and the exact position of each single grain from the confocal pictures. A regression equation from the results of single-grain scratch tests was determined and applied to the simulation model. The developed model enabled the prediction of the micro-grinding forces and surface roughness. The simulated forces and surface roughness were compared to the experimental results. Both the simulated forces and surface roughness followed the same trends as the empirical results. Increasing the cutting speed decreased the grinding forces in both simulation and experimental results. A similar influence was seen for varying depth of cut. The higher depth of cut increased the grinding forces and surface roughness values in both simulation and experimental results. However; both empirical and simulation results showed that the feed rate does not influence the surface roughness significantly. The model predicted the mean tangential and normal grinding forces with a total error of 13.5 %. In the case of prediction of the ground surface roughness, although, the simulation results were matched with the experimental results in greater degree in the low cutting speed of 6m/s, rather in the higher cutting speed (14m/s) because the tool
vibration which occurs at the higher cutting speeds was not considered in the model. The surface roughness was predicted at the cutting speed of 6 m/sec with the total error of 16 percent among the mean values.

The model shows that the effects of process parameters and wheel surface conditions could be acceptably mapped to the process outputs, of course with a degree of accuracy and simplification. **The modeling process** can be applied, if the tool conditions (initially or after being worn) could be described as a function of the grain size, concentration, height and distribution. Further robustness tests of the model for different conditions and material are currently being performed.

**References**


