Stochastic modelling of 3D surface topography for coated abrasive discs

Adhithya Plato Sidharth Arunachalam\textsuperscript{1,a*}, Idapalapati Sridhar\textsuperscript{1,b} and Takashi Sato\textsuperscript{2,c}

\textsuperscript{1}School of Mechanical & Aerospace Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798, Singapore
\textsuperscript{2}Advanced Remanufacturing and Technology Centre, Agency for Science, Technology and Research (A*STAR), 3 CleanTech Loop, Singapore 637143, Singapore

\textsuperscript{a}adhithya002@e.ntu.edu.sg, \textsuperscript{b}msridhar@ntu.edu.sg, \textsuperscript{c}satot@artc.a-star.edu.sg

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Abstract. A stochastic model is developed for coated abrasive disc with precision prism-shaped grains. Parameters like protrusion height, inter-grain spacing and grain density are usually considered for modelling 3D abrasive surface topography. In this study, in addition to these existing parameters, the final surface topography of the #60 grit coated abrasive disc is generated considering the orientation angles of the grains. The grains are rotated iteratively by using randomly generated pitch, roll and yaw angles until the grains protrusion height agreed with the experimental distribution. The protrusion height and inter-grain spacing distribution parameters, 3D height parameters ($S_q$, $S_{sk}$, $S_{ku}$) and spatial parameters ($S_{al}$, $S_{tr}$) of the simulated surface are in good agreement with the measured surface.

Introduction

Coated abrasive discs are used in manual or automated (using robots) finishing operations in precision manufacturing industries. The coated abrasive tools are compliant in nature and are preferred compared to the grinding wheels because of (i) finer material removal (ii) ability to adapt to the complex surfaces. The coated abrasive disc consists of backing cloth or paper, followed by a layer of resin on which grains are placed randomly and held together by size coat resin layer as shown in Fig.1

![Fig.1 Schematic representation of conventional coated abrasive disc [1]](image)

The material removal depth calculations in polishing inherently use the contact pressure variation over the grain distribution [2]. Hence, the first step in material removal modelling in polishing with coated abrasive disc is grain distribution model. The existing topography models can be broadly classified into one, two and three-dimensional models, each giving different levels of information. Although they are similar in most of the aspects, in the case of 3D models the grains, the spacing between the grains and also the resultant surface is three dimensional [3]. Hegeman et al. [4] considered parameters like protrusion height, inter-grain spacing for modelling the three-dimensional surface by assuming the grains in ellipsoidal shape. Feng et al. [2] developed a similar 3D surface for cone shaped grains, and additional resin layer was modelled to replicate the actual surface. Woodin [5] modelled the grains as 3D polygons with different sides based on the statistical
information obtained from the actual grain aspect ratio, facets, size, and protrusion height. Unlike the conventional coated abrasive discs where the grains are irregular in shape, uniformly shaped grains are comparatively durable and have better performance. Although the shape of the grains is fixed, yet the protrusion height is still random in nature. This could be attributed to either the placement of the grains within the resin layer of the coated abrasive disc or due to the randomness of grain orientations caused while depositing the grains on the make coat.

In the current investigation, a three-dimensional surface topography model for #60 grit size coated abrasive disc with precision shaped (3M® Cubitron II) by the stochastic process is proposed where parameters like protrusion height, inter-grain spacing and additionally grain orientations are considered. The simulated surface is validated with the measured surface results using the 3D surface parameters.

**Measurement results**

The #60 grit coated abrasive disc is scanned with 50 µm resolution using a laser probe in Talyscan profilometer. As shown in Figs. 2(a) and 1(b), 10 mm x 10 mm scan area able to comprehensively depict the grain density (of topography in Fig. 2(a)).

![Fig 2 (a) Measured 3D surface, (b) scanned area versus grain density](image)

Generally, abrasive surface can be divided into low, medium and high frequency components corresponding to waviness, grains and noise respectively. Using Fourier transform filtering technique, the high frequency component is removed [6] and the filtered surface is obtained as shown in Fig. 3(a). The peaks are extracted using the code by finding the maximum value from the eight surrounding cells. The dimensions of the grains are obtained from the micrograph as shown in Fig. 3(b). The triangular prism shaped grain has three equal sides of 0.63 mm ($L_g$) and 0.12 mm ($W_g$) width.

![Fig. 3 (a) Filtered surface with peaks & valleys, (b) typical abrasive grain geometry with dimensions.](image)
Extracting statistical information

Protrusion height is the height of the peaks from the resin layer. From the cross-section image of the coated abrasive disc, the average height of the size coat is measured to be 0.2 mm. This threshold distinguishes the peaks of grains and the size coat. Also, the grains which are less likely to participate in material removal is excluded. Using the programme code, the protrusion peak height is obtained and found to follow normal distribution (Fig. 4(a)) which agrees with the existing works [7, 8]. The normal distribution is confirmed by Anderson-Darling test at 95% confidence level. In order to calculate the inter-grain distance, the scanned area is segmented into lattices with $\left(\sqrt{S_g}\right)^{-1}$ [4]. With reference to the grain in the centre cell, the average distance is calculated from the grains in the surrounding eight cells. The inter-grain spacing follows close to normal distribution as shown in Fig. 4b. The mean and standard deviation values for inter-grain spacing and protrusion peak height are tabulated in Table 1: there is a good comparison between the experiment and simulation.

Fig. 4 Experimentally measured (a) protrusion peak height, (b) inter-grain spacing distribution

Stochastic modelling of the 3D surface

The flow-chart in Fig. 5 outlines the modelling approach adopted in the investigation.

Fig. 5 Flowchart for stochastic simulation of coated abrasive
Spatial distribution and peak height simulation

The grains are randomly distributed using the method called ‘random grain shaking’ [9]. Initially, the grains are arranged in the lattice centre with dimensions as discussed earlier. The new spatial coordinates are obtained from Eq. 1.

\[
\begin{bmatrix}
    x'_{cij} \\
    y'_{cij} \\
    z'_{cij}
\end{bmatrix} = \begin{bmatrix}
    x_{cij} + \delta_x \\
    y_{cij} + \delta_y \\
    z_{cij} + \delta_z
\end{bmatrix}
\]  

where \(i=1, 2, \ldots, M\) and \(j=1, 2, \ldots, N\)

\(M, N\) is the number of rows and columns of the area respectively. Suffix ‘c’ indicates the position of lattice centre and ‘g’ transformed coordinate. \(\delta_x, \delta_y\) are random numbers from normal distribution \(\delta_z\) is displacement replicating grain position within the resin layer which follows normal distribution.

In order to simulate the protrusion height, the orientation angle of the grains is also considered. Each grain will be rotated according to the roll, pitch and yaw angles about the origin and translated to the respective positions as shown in Fig. 6(b). The orientation angles are randomly generated from a uniform distribution with range \(\pm 45^\circ\). The grain vertices are transformed using rotation and translation matrix. The detailed description of the rotation matrix can be found elsewhere [10]. The iterations are carried out till the simulated peak distribution (Fig. 6(a)) matches the experimentally measured distribution parameters. The mean and standard deviation of the simulated inter-grain spacing and protrusion peak height is compared with the actual specimen in Table 1.

![Simulated protrusion peak height](image1)

![Randomly oriented grains](image2)

Table 1 Comparison between experimental and simulation distributions

<table>
<thead>
<tr>
<th>Statistical parameters</th>
<th>#60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean protrusion height ((\mu_{ph})) (mm)</td>
<td>0.344</td>
</tr>
<tr>
<td>Standard deviation protrusion height ((\sigma_{ph})) (mm)</td>
<td>0.058</td>
</tr>
<tr>
<td>Inter-grain spacing ((\mu_{igs})) (mm)</td>
<td>1.158</td>
</tr>
<tr>
<td>Standard deviation inter-grain spacing ((\sigma_{igs})) (mm)</td>
<td>0.105</td>
</tr>
</tbody>
</table>

In order to generate the surface replicating the scanned abrasive surface, bi-harmonic surface fitting is carried on the simulated 3D grains using MATLAB® toolbox [11]. The final simulated surface is shown in Fig. 7.
The experimental and simulated surface are compared using the 3D surface parameters such as $S_q$, $S_{sk}$, $S_{ku}$, $S_{dq}$ and spatial parameters like $S_{tr}$ and $S_{al}$ using the program. The simulated surface is validated against the measured surface using the 3D surface parameters as listed in Table 2: experimentally measured parameters agree quite well with the numerical simulations.

Table 2 Validation of 3D surface parameter

<table>
<thead>
<tr>
<th>3D surface parameters</th>
<th>#60</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
</tr>
<tr>
<td>$S_q$</td>
<td>0.503</td>
</tr>
<tr>
<td>$S_{sk}$</td>
<td>0.251</td>
</tr>
<tr>
<td>$S_{ku}$</td>
<td>0.099</td>
</tr>
<tr>
<td>$S_{dq}$</td>
<td>0.37</td>
</tr>
<tr>
<td>$S_{al}$</td>
<td>0.85</td>
</tr>
<tr>
<td>$S_{tr}$</td>
<td>0.84</td>
</tr>
</tbody>
</table>

**Conclusion**

A three-dimensional surface topography modelling technique is proposed for precision-shaped grains by considering random grain orientations. Coated abrasive disc with triangular prism shaped grains is scanned using a laser probe, and the protrusion peak height and inter-grain spacing followed normal distribution. Using these statistical parameters and grain dimensions, 3D grains are modelled and randomly scattered matching the experimentally measured distribution. The grains are randomly oriented using the roll, pitch, and yaw angles iteratively such that the experimental peak distribution is matched. From the generated 3D grains, the surface topography of coated abrasive disc is created by finally using bi-harmonic surface fit. The simulated surface shows good agreement with the actual sample’s 3D surface parameters.

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References


